Absolute Value Equations and Inequalities

equations $a|bx+c| = d$

$$|bx+c| = \frac{d}{a}$$

special case $|x| = -\#$ → no soln

break this up into two equations and solve each one.

$$bx+c = \frac{d}{a} \qquad bx+c = -\frac{d}{a}$$

$$bx = \frac{d}{a} - c \qquad bx = -\frac{d}{a} - c$$

$$x = \frac{1}{b}\left(\frac{d}{a} - c\right) \qquad x = \frac{1}{b}\left(-\frac{d}{a} - c\right)$$

less thans — intersections — logical "and"

(usually the result is an interval like ○——○)

special case $|x| < -\#$ → no soln

Ex $2|x-4| + 6 < 10$

$2|x-4| < 4$

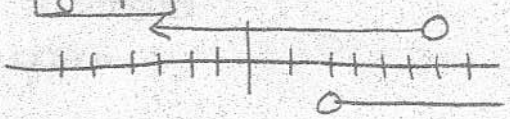
$|x-4| < 2$

break it up into two inequalities

$x-4 < 2$ and $x-4 > -2$

$x < 6$ $\qquad x > 2$

graph



interval notation $(2, 6)$

short cut for less thans

$$2|x-4|+6<10$$
$$2|x-4|<4$$
$$|x-4|<2$$

Interval notation $(2,6)$

$$-2<x-4<2$$
$$\boxed{2<x<6}$$

greater thans

unions — logical "or"

result is usually

$(-\infty, a) \cup (b, \infty)$

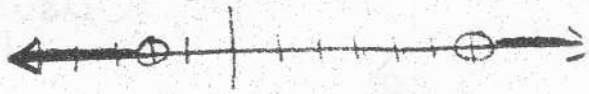
special case

$|x| > -\#$

all real numbers

Example

$$3|x-2|-4>8$$
$$3|x-2|>12$$
$$|x-2|>4$$



$(-\infty, -2) \cup (6, \infty)$

Interval notation

break it up into two inequalities

$$x-2>4 \quad \text{or} \quad x-2<-4$$
$$x>6 \quad \text{or} \quad x<-2$$